A SERIES RESONANT CRYSTAL CONTROLLED OSCILLATOR

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A SERIES RESONANT CRYSTAL CONTROLLED

OSCILLATOR

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INTRODUCTION

The use of quartz crystal resonators to provide a high degree of frequency stability in oscillators is a well recognized and widely used practice. The excellent frequency control which a crystal exhibits is due to its extremely high Q as a mechanical resonator and, through the piezoelectric effect, as an electrical resonator.

The resonances of a crystal are actually mechanical in nature. Thus, it is evident that the crystal is capable of resonance at many different frequencies depending upon the method of excitation. A practical crystal unit will have a nominal frequency at which it will resonate in an oscillator. Although it may have resonant responses at other frequencies, the characteristics of these responses are such that they are considered spurious and, generally speaking, may be ignored when the crystal is used in an oscillator. It has been shown by K. S. Van Dyke that a crystal operating at or near its principal resonance can be accurately represented by the equivalent electric circuit shown in Fig. 1. The capacitance, C_h, represents the capacity between the crystal holder plates with the crystal as a dielectric, while L_o, R_o, and C_o are the electrical equivalents of the mechanical properties, mass, friction, and elasticity, respectively. Various investigators have determined representative values for the equivalent circuit; typical values being $L_o = 1/16$ henry, $R_o = 30$ ohms, $C_o = 0.04$ micro-farads, and

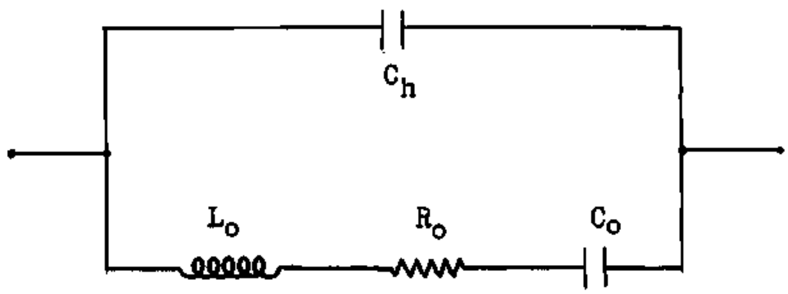

Fig. 1. Equivalent Circuit for a Crystal Near Resonance.

C_h = 28 micro-micro-farads. This example has a nominal resonant frequency of 3.2 megacycles. It is immediately apparent from these element values that this device has an extremely high Q.

Further examination of the above circuit reveals that it has both resonant and antiresonant properties. Thus a crystal in the region of the so-called "nominal" resonant frequency has a very low impedance at a certain frequency and a very high impedance at a slightly higher frequency. These two frequencies are separated by an extremely small amount. The resistive and reactive components of the crystal impedance are shown on an expanded frequency scale in Fig. 2. The magnitudes of the components are such that the phase angle as a function of frequency changes very rapidly in the regions of resonance and antiresonance. Apparently this fact was overlooked by many of the early investigators.

The more commonly used oscillator circuits such as the Pierce and Miller circuits operate the crystal on the point where the phase shift is appreciable. As a result, the frequency of oscillation is appreciably affected by the variations in the oscillator circuit impedances due to voltage fluctuations, non-linearity, component ageing, and various other random effects. It might be said that these circuits

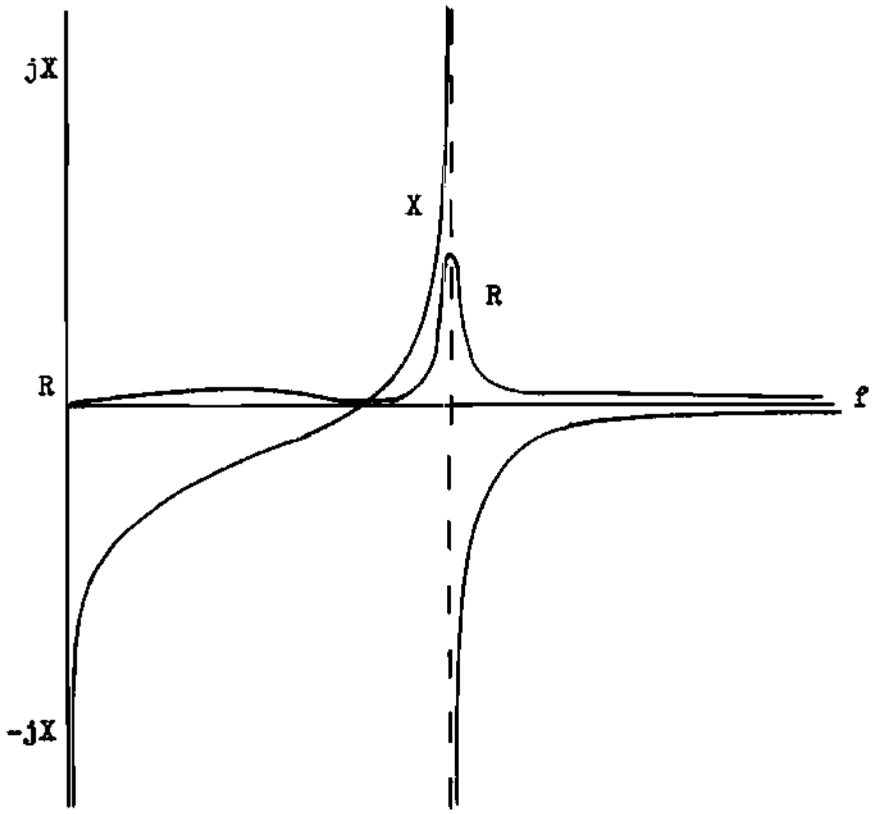

Fig. 2. Resistive and Reactive Components of Crystal Impedance as a Function of Frequency.

operate the crystal at a type of antiresonance where the variations in impedances shunting the crystal would have an appreciable effect on the frequency. However, if the crystal were operated very near series resonance, the frequency effects of these stray shunt impedances would be minimized since the crystal itself has such a low impedance at series resonance. This evident advantage of series resonant crystal operation has been recognized by several investigators and used to produce better frequency stability.

Since crystal oscillators invariably employ vacuum tubes, it is necessary to consider the limitations and requirements imposed by the vacuum tube used in the oscillator circuit. In practical circuits, tubes usually operate best with plate impedance loads around 10,000 ohms. On the other hand, the average crystal will have an impedance of 100 ohms at series resonance and 5 megohms at antiresonance. It is immediately evident that oscillator circuits which use the crystal directly with the tube do not satisfy the impedance requirements of either the crystal or the tube. It appears that some method of impedance transformation which will provide appropriate impedance levels for both the crystal and tube is required if optimum circuit performance is to be obtained.

It is desired to devise and investigate an oscillator circuit which will take advantage of the properties of a crystal operating at series resonance and also produce proper impedance levels for both the crystal and the vacuum tube. An oscillator which makes use of transformers will permit simultaneous fulfillment of both of these objectives. At this point it should be noted that the oscillator must not be capable of operation in the absence of the crystal or if the crystal breaks. In addition, the circuit must be capable of functioning with crystals covering a band of frequencies.

THE BASIC CIRCUIT AND CONDITIONS FOR OPERATION

Quite evidently for a single-tube oscillator, two transformers are required. Furthermore, the position of the crystal in the circuit must take advantage of the series resonant properties of the crystal. Fig. 3 presents an example of such a transformer-coupled oscillator.

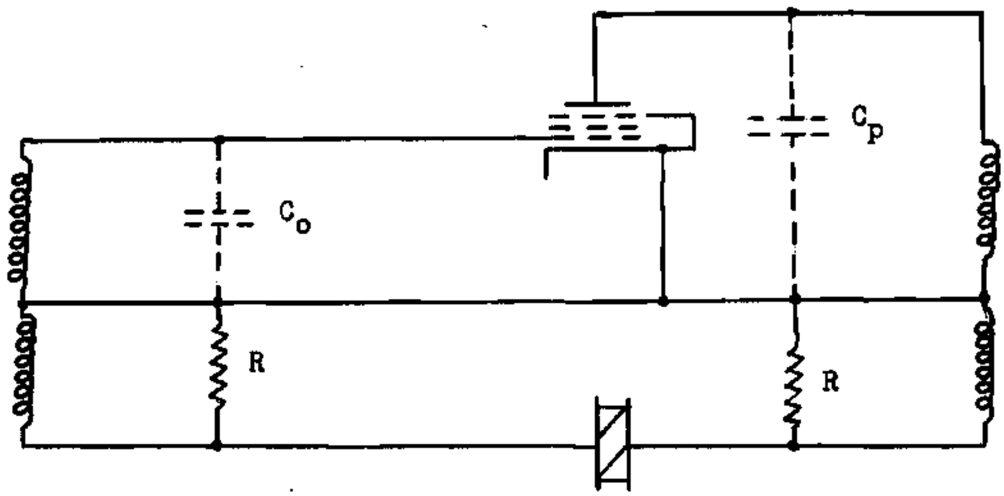

Fig. 3. Transformer Coupled Oscillator.

If the resistances, R, are low (of the order of the series resonant crystal impedance) the feedback path through the crystal will permit oscillations to be sustained only at frequencies very near the crystal resonance. Furthermore, the circuit is incapable of oscillating in the absence of the quartz plate. Limitations on this type of circuit are imposed by the available tubes, parasitic capacities, and the transformers.

The circuit of Fig. 3, composed of transformers, capacitances, and resistances, may be represented by a pair of general 4-terminal

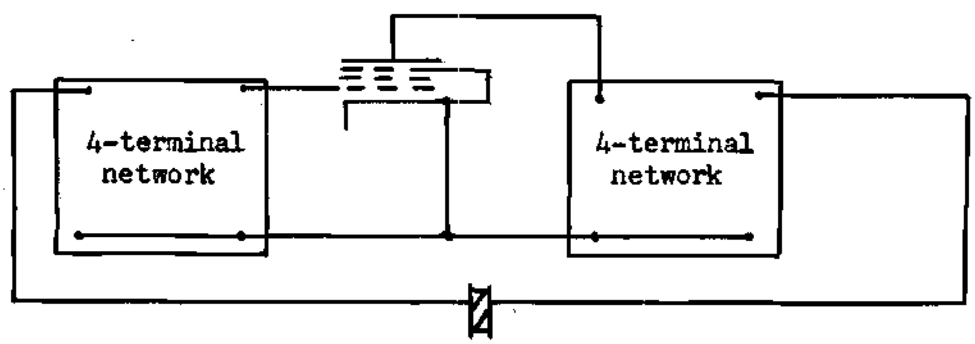

Fig. 4. Equivalent Circuit of Fig. 3.

networks and an ideal tube, as shown in Fig. 4. The analysis is simplified by initially assuming that the two networks are identical. True series resonant operation requires that the crystal be a pure resistance: that is, there must be no phase shift through the crystal. To approximate this ideal condition, the networks must produce negligible phase shift over the frequency band for which the oscillator is designed. The requirements on the gain characteristic are not so severe. Sufficient gain must be available over the band to cause oscillations to build up, but excessive gain at certain portions of the band is not detrimental. However, the gain should decrease to values which are insufficient to sustain oscillations at frequencies outside the operating band.

It has been shown by Bode[1] that any low pass network can be transformed to a band pass network, and that knowledge of the characteristics of the low pass network permits calculation of the band pass network characteristics. The chief advantage of this method is that the behavior of low pass networks is more familiar and is more easily studied than that of corresponding band pass structures. In terms of a low pass network, the requirement is that there shall be negligible phase shift and relatively constant gain from zero frequency up to some limiting frequency. Above this frequency there is no specific requirement on either characteristic; however, in practice, the gain will ultimately fall off and the phase will approach some small integral multiple of 90°. One convenient idealized phase characteristic

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[1] H. W. Bode, Network Analysis and Feedback Amplifier Design (New York: Van Nostrand Co., Inc., 1945), p. 210 ff.

is shown in Fig. 5.

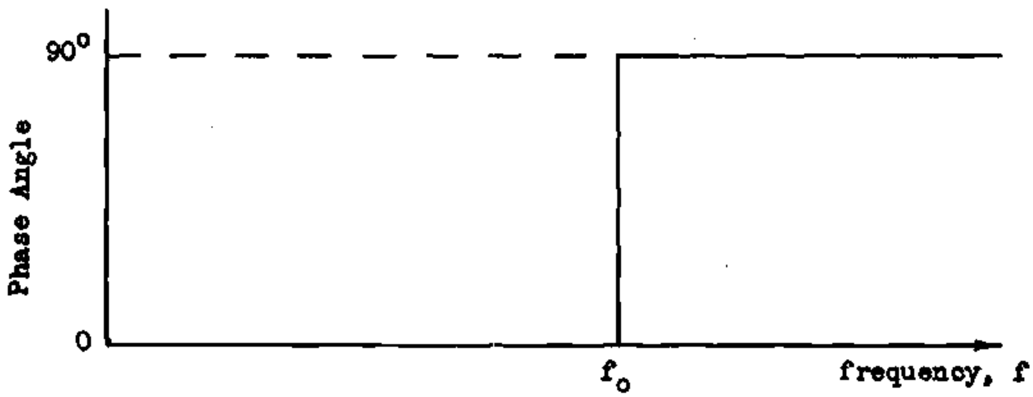

Fig. 5. Idealized Phase Characteristic for Low Pass Network.

Bode's investigations show that the gain and phase characteristics are interdependent for a minimum phase shift network; furthermore, all passive ladder networks are of the minimum phase type.[2] Therefore, for any specified phase characteristic of a ladder network there is only one possible gain characteristic. Fig. 6 shows the gain characteristic which corresponds to the phase characteristic of Fig. 5. These gain and phase relationships are for the transmission characteristics of a specific 4-terminal low pass network. They represent only one of many possible engineering solutions to the problem. It should be noted that the gain and phase curves are proportional to each other, so that one will be doubled if the other is doubled.

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[2] The definition of "minimum phase shift" networks and some material especially pertinent to this discussion are given on p. 121 and 242 ff. of Bode.

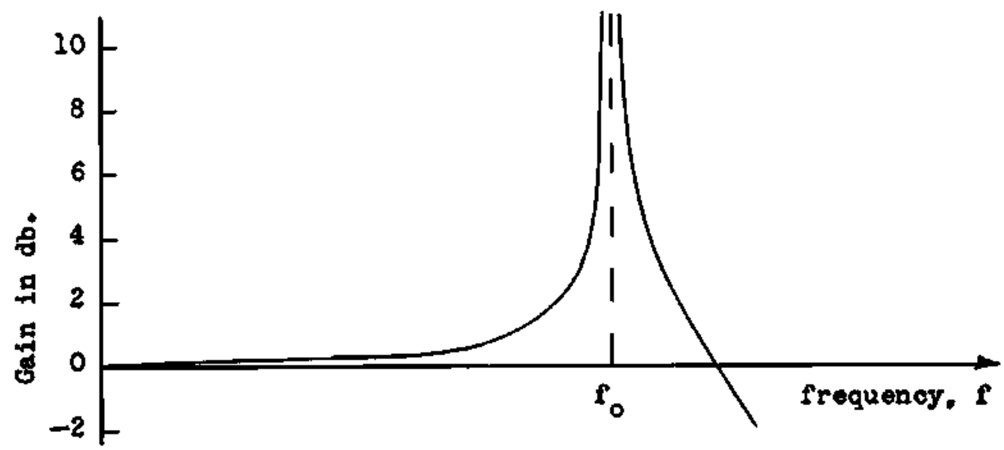

Fig. 6. Associated Gain Characteristic
for Low Pass Network.

Development of a suitable low pass network does not constitute a solution to the problem, because there is the additional requirement that the network be capable of transformation to a band pass impedance-transforming network by the use of physically realizable elements. In particular, the required transformers must have inductance values and coefficients of coupling which can be realized over the required frequency band.

SYNTHESIS OF A SUITABLE NETWORK

In the equivalent circuit of Fig. 4 the right-hand 4-terminal network is driven by a pentode, which is assumed to have an infinite dynamic plate resistance and is, therefore, a constant-current generator. The amplifying property of the tube is unavoidably accompanied by capacitance, which is indicated in Fig. 7 as being in shunt with terminals 1 and 2 of the network. The tube may now be regarded as an ideal constant-current generator. It can also be shown that corresponding to a practical transformer with a realizable coefficient of coupling in the band pass network, the low pass network must have an unshunted series inductance between terminals 1 and 3. This inductance is equivalent to the leakage reactance of the transformer. Thus, the transfer phase and gain characteristics and two elements of the network are known. Fig. 7 shows the partially determined 4-terminal network.

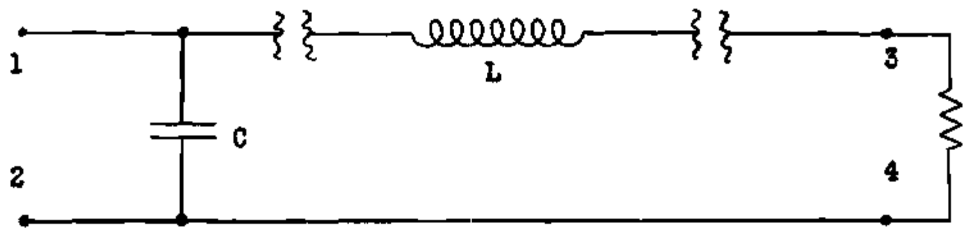

Fig. 7. Minimum Partial Low Pass Network from Practical Considerations.

Deduction of the remaining network elements involves design practices which are not unique. It is therefore possible that the network derived in the following paragraphs is not optimum in either performance or simplicity.

The design offered is based upon the gain characteristic of Fig. 6, from which it is seen that the gain increases with frequency up to the point f_o. One network completion of Fig. 7 which approximates that characteristic is shown in Fig. 8. The phase characteristics of the transfer impedance of this network are presented in a normalized form in Fig. 9. In this normalized presentation the parameters L_1, L_2, and R_2 are related to R_1 and C_3 by the following factors:

$$L_1 = h^2 C_3 R_1 , \quad (1)$$

$$L_2 = p^2 C_3 R_1 , \quad (2)$$

$$R_2 = r\, R_1 , \quad (3)$$

and

$$f_o = \frac{1}{2\pi C_3 R_1} . \quad (4)$$

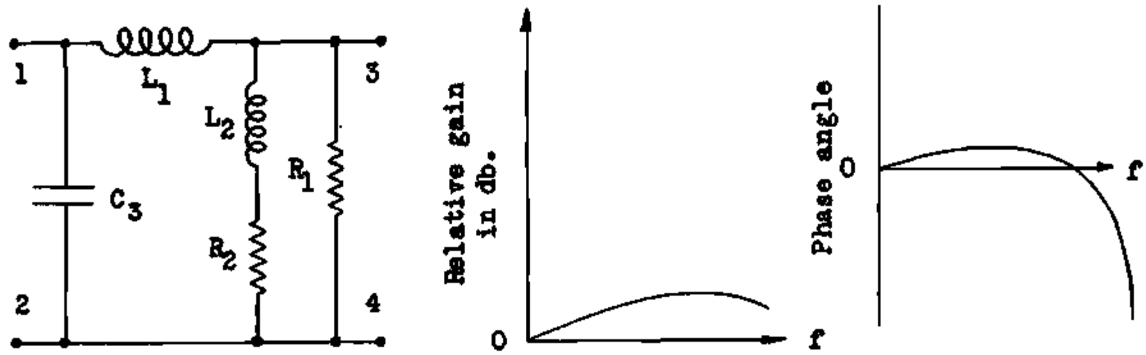

Fig. 8. A Possible Solution for Desired 4-Terminal Low Pass Network.

The value of R_1 is fixed approximately by the tube and the gain requirements, and the minimum C_3 is fixed by the tube. The effects of varying L_1, L_2, and R_2 are determined by varying the factors h, p, and r. From the curves of Fig. 9, it is seen that changing h affects the bandwidth achieved for a given maximum phase shift. In addition, it will be found that h also controls the coefficient of coupling required in a practical transformer. Extensive calculations have indicated that with a fixed h the maximum bandwidth which can be obtained for a given maximum allowable phase shift is determined quite accurately by the equation

$$p' = r\, x \text{ (a constant)} . \quad (5)$$

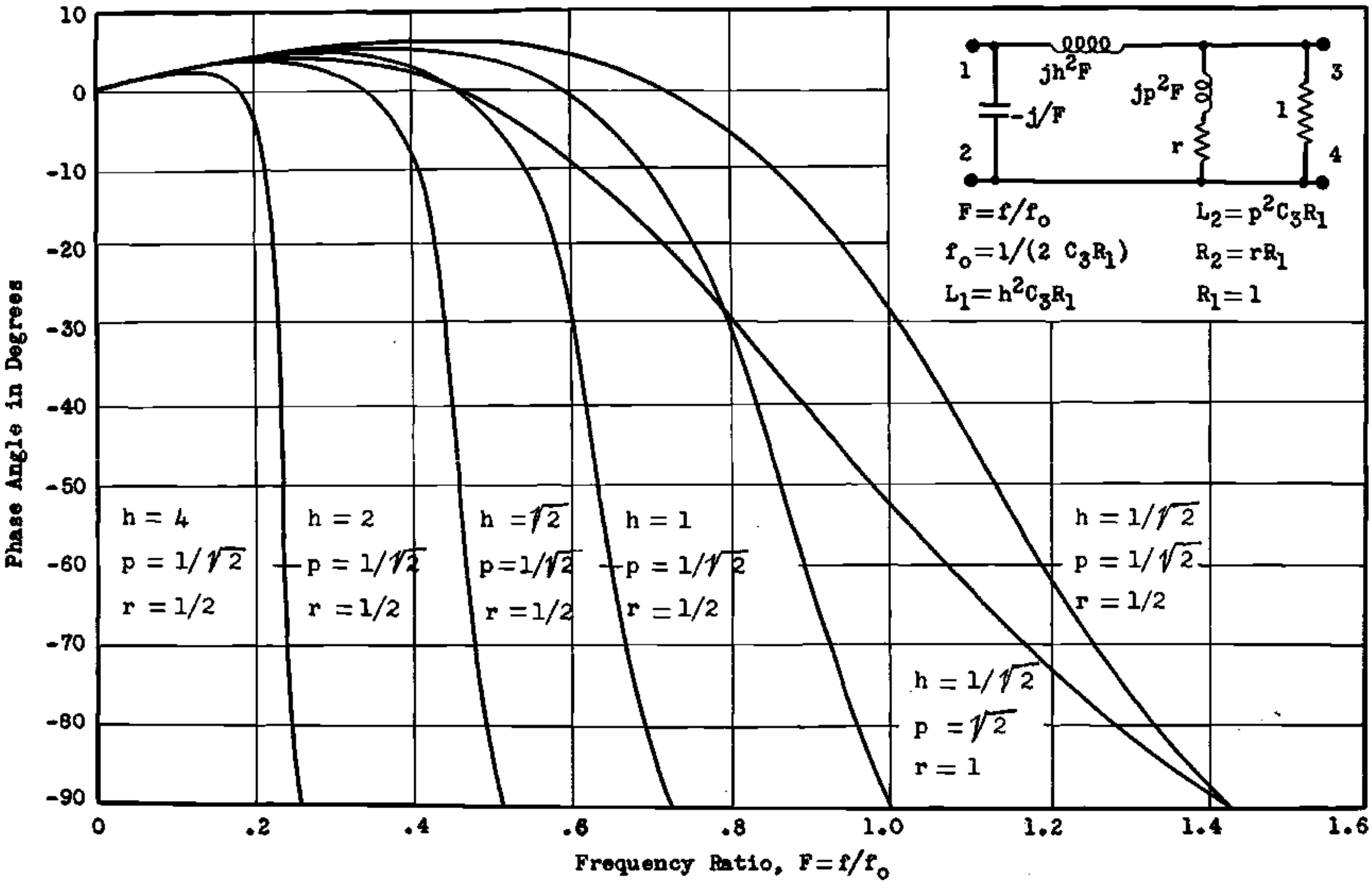

Fig. 9 - Normalized Transfer Characteristics for Network of Fig. 8.

Since it is desired to operate the crystal very close to series resonance, a maximum phase shift of $\pm\ 5^{\circ}$ is arbitrarily specified. For a maximum phase shift of $\pm\ 5^{\circ}$ the constant in (5) is approximately $\sqrt{2}$, so that

$$p = r\sqrt{2} \ . \tag{6}$$

DETERMINATION OF CIRCUIT GAIN REQUIREMENTS

Because the phase characteristic of the network is known, certain deductions and restrictions may be made concerning the gain characteristic. In the process of transforming a low pass network into a band pass network, the gain of the low pass network at zero frequency becomes the gain of the band pass network at its mean frequency. It can be shown that when the phase limitations are imposed, the gain of the network of Fig. 8 is a minimum at zero frequency. Let us represent an equivalent crystal impedance (which will be essentially resistive when operated properly) by a resistance, R_x. The low pass equivalent of the oscillator is illustrated in Fig. 10.

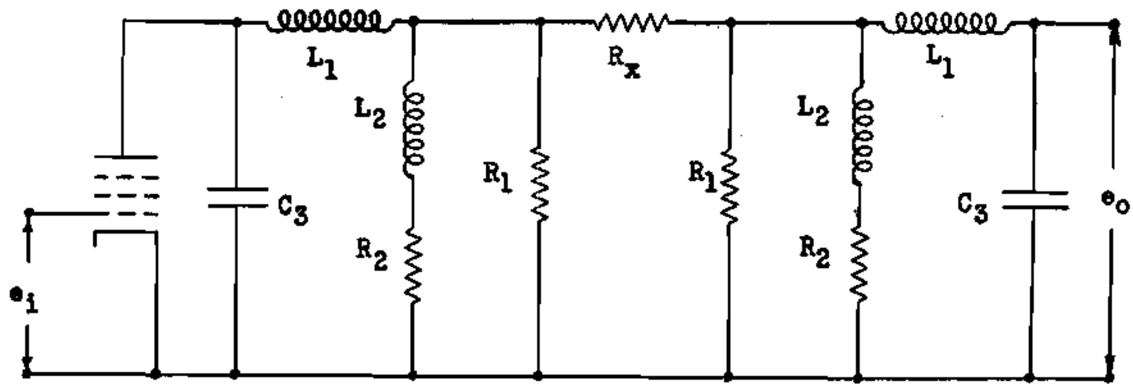

Fig. 10. Low Pass Network Equivalent of Transformer Coupled Oscillator of Fig. 3.

Calculations show that for a prescribed phase characteristic and overall gain at zero frequency, the bandwidth of this low pass equivalent is relatively independent of r ($= R_2/R_1$). Good oscillator performance is anticipated if $R_x = R_1$. By making the restriction that $R_1 = R_2 = R_x$ the equivalent circuit of Fig. 11 is obtained for frequencies near zero.

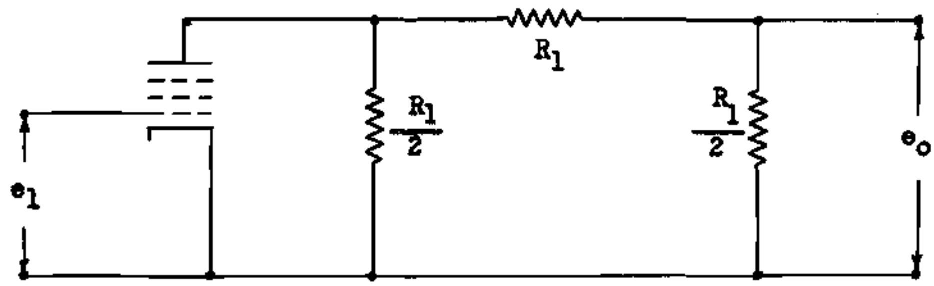

Fig. 11. Low Pass Equivalent of Fig. 10
Near Zero Frequency.

The overall gain at low frequency is given by

$$\text{Gain} = \frac{e_o}{e_1} = \frac{R_1 g_m}{8} \quad . \tag{7}$$

To provide a margin for operation, let us specify that the gain shall be 2. The resistance is than fixed by the relationship

$$R_1 = 16/g_m \quad . \tag{8}$$

TRANSFORMATION OF THE LOW PASS NETWORK
TO AN IMPEDANCE TRANSFORMING NETWORK

The transformation of the network in Fig. 8 to a band pass network is accomplished by adding an inductance in shunt with each shunt capacitance and a capacitance in series with each series inductance in the original network. The added parameters are selected to resonate or antiresonate with their complementary parameters at a frequency which is the geometric mean of the band to be covered. This transformation is shown in Fig. 12. The values for the various dependent parameters are given by the equations

$$L_3 = \frac{1}{4\pi^2 f_m{}^2 C_3}, \tag{9}$$

$$C_1 = \frac{1}{4\pi^2 f_m{}^2 L_1}, \tag{10}$$

and

$$C_2 = \frac{1}{4\pi^2 f_m{}^2 L_2}, \tag{11}$$

where f_m is the geometric mean frequency of the band.

Certain arbitrary restrictions have been imposed on the various parameters. Since $R_1 = R_2$, then $r = 1$ by (3). Also, using (1) and (8), (10) becomes

$$C_1 = \frac{g_m}{64\,\pi^2 f_m{}^2 h^2 C_3}. \tag{12}$$

Finally, when $r = 1$, (6) yields $p = \sqrt{2}$. Taking this value in conjunction with (2) and (8), (11) becomes

$$C_2 = \frac{g_m}{128\,\pi^2 f_m{}^2 C_3}. \tag{13}$$

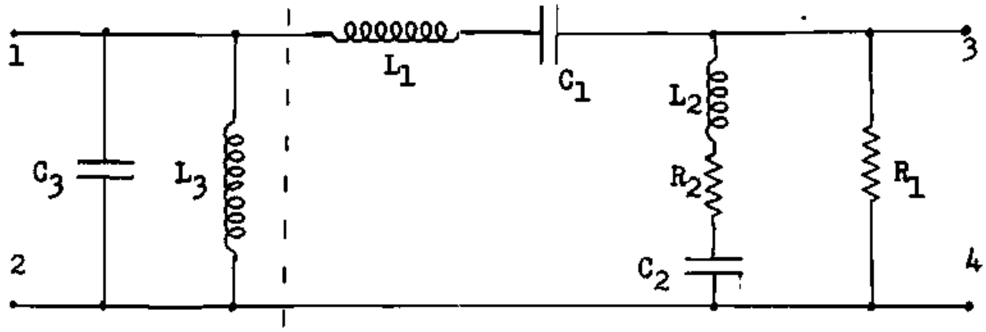

Fig. 12. Band Pass Network Corresponding
to Low Pass Network of Fig. 8.

The band pass network is converted into an impedance transforming network by inserting an ideal transformer at the dotted line in Fig. 12. This transformer has an impedance ratio of ϕ^2 and is step-down from left to right. By defining $\phi^2 < 1$ the network of Fig. 13 is obtained. By well known processes[3] the elements between a-b in Fig. 13

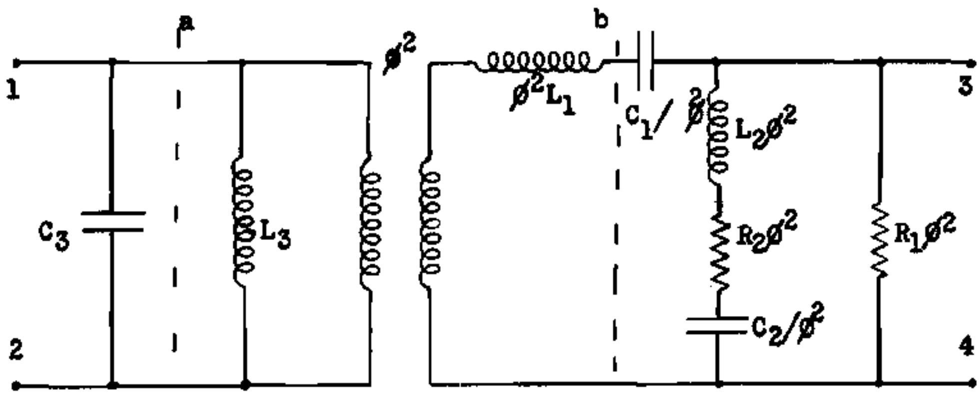

Fig. 13. Impedance Transforming
Band Pass Network.

can be obtained by employing a practical transformer as shown in Fig. 14.

[3] T. E. Shea, Transmission Networks and Wave Filters (New York: Van Nostrand Co., Inc., 1929), p. 325 ff.

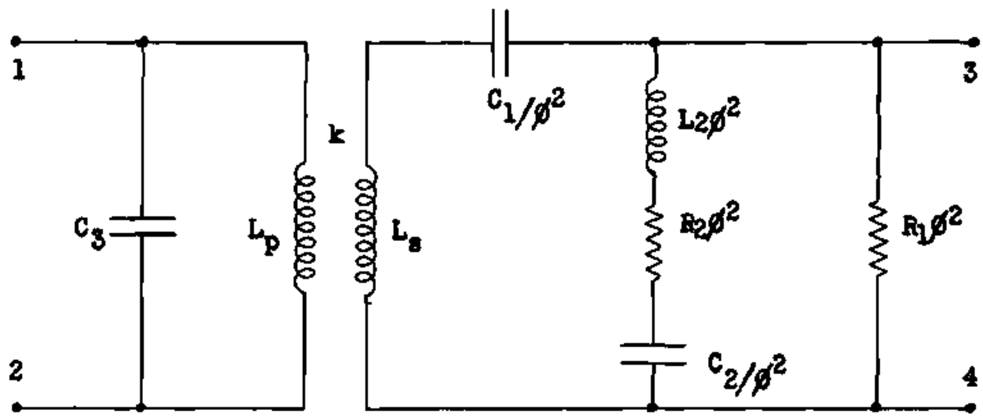

Fig. 14. A Practical Network.

The values for L_p, L_s, and k, the coefficient of coupling, are given by

$$L_p = L_3 , \tag{14}$$

$$L_s = \phi^2 (L_1 + L_3) , \tag{15}$$

and

$$k = \sqrt{\frac{L_3}{L_1 + L_3}} . \tag{16}$$

Because the restriction $R_1 = R_x$ was imposed, the value for ϕ^2 can be determined. The impedance of the crystal at series resonance closely approximates the R_o of the crystal. Therefore, it is desired that ϕ^2 have a value such that

$$\phi^2 R_1 = R_o ,$$

or from (8)

$$\phi^2 = \frac{R_o g_m}{16} . \tag{17}$$

We may now specify the values for the various oscillator components, shown in Fig. 15, in terms of C_3, g_m, f_m, and <u>h</u>.

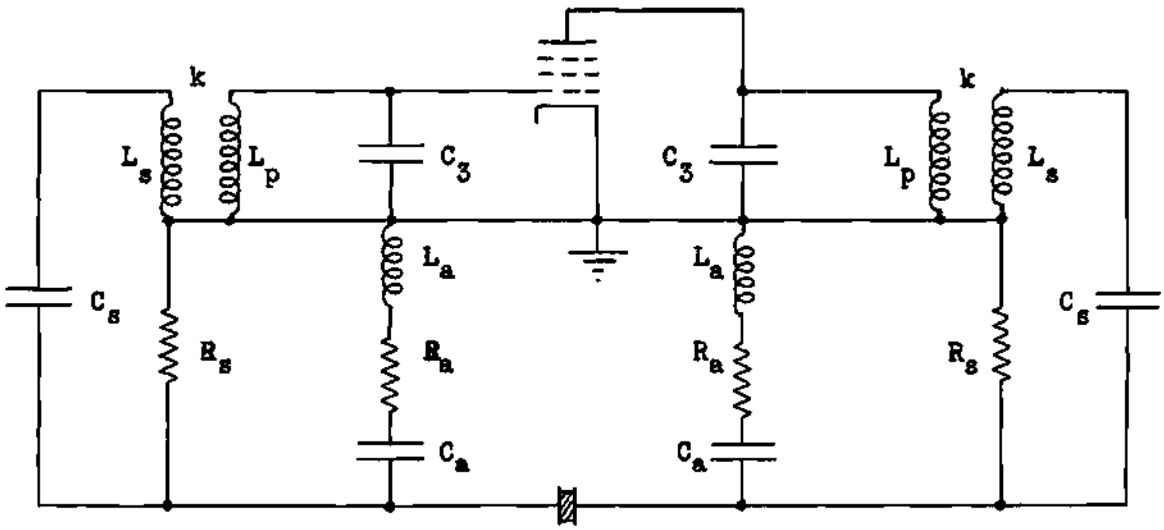

Fig. 15. Basic Oscillator.

$$L_p = \frac{1}{4\pi^2 f_m^{\,2} C_3} , \tag{18}$$

$$L_s = \frac{R_o g_m}{16}\left[\frac{1}{4\pi^2 f_m^{\,2} C_3} + \frac{16h^2 C_3}{g_m}\right] , \tag{19}$$

$$C_s = \frac{1}{4\pi^2 f_m^{\,2} h^2 C_3 R_o} , \tag{20}$$

$$R_s = R_o , \tag{21}$$

$$R_a = R_o , \tag{22}$$

$$C_a = \frac{1}{8\pi^2 f_m^{\,2} C_3 R_o} , \tag{23}$$

$$L_a = 2C_3R_o , \tag{24}$$

and

$$k^2 = \frac{1}{1 + \dfrac{64\pi^2 f_m{}^2 C_3{}^2 h^2}{g_m}} . \tag{25}$$

This analysis is based on the assumption that the grid and plate transformers are identical. If the tube imposed the only limitation on the performance of the oscillator, it can be shown that the input and output admittances of the vacuum tube are the limiting factors. However, since the input and output admittances of a typical pentode oscillator are approximately equal, the required transformers are essentially the same, and the initial assumption is reasonable.

DISCUSSION OF A PRACTICAL OSCILLATOR

It has been found that $\underline{k}$ is the factor which limits wide band operation in the frequency range from 1 to 8 mc. Actually, for reasonable values of $\underline{h}$, C_3, and g_m, $\underline{k}$ must be greater than 0.8 if wide band operation is to be obtained in this frequency range. Specific calculations for an assumed $k = 0.5$ gives a band about 25 kc wide for which the phase shift does not exceed $\pm 5^o$ in each transformer. This calculation is illustrated below.

Assume: $k = 0.5$, $h = 4$, $g_m = 10{,}000$ micromhos, and $f_m = 2$ mc. Then from (25)

$$C_3{}^2 = \frac{(1 - k^2)g_m}{64\pi^2 f_m{}^2 h^2 k^2} . \tag{26}$$

The numerical solution is

$$C_3^2 = \frac{(1 - .25)\ (.01)}{64\pi^2\ (2 \times 10^6)^2\ (4)^2\ (.5)^2} = .74 \times 10^{-18} \quad ,$$

or

$$C_3 = 860\mu\mu f \ .$$

This value gives, by (4) and (8), a "critical" frequency of

$$f_o = \frac{g_m}{32\pi C_3} \ ,$$

or

$$f_o = \frac{.01}{32\pi(860 \times 10^{-12})} = 115 \text{ kc.}$$

Referring to Fig. 9, it is seen that for h = 4 the frequency at which the phase deviates $\pm\ 5^{\circ}$ is

$$f' = .2f_o = 23 \text{ kc.}$$

But the bandwidth of the low pass and band pass structures are the same, and so this is the band over which the oscillator will operate when referred to f_m. That is,

$$\text{Operating Band} = f_m \pm f'/2 \ ,$$

$$= (2 \pm .0115) \text{ mc.}$$

When the coefficient of coupling, <u>k</u>, exceeds about 0.9, the tube capacitances limit the obtainable bandwidth. Whether such close coupling can be obtained with existing transformer construction techniques is not known. If it can, comparatively wide band operation may be secured in the frequency range from 1 to 8 mc.

The use of a pentode is mandatory because of the initial requirement that the circuit be incapable of operation in the absence of the crystal. Essentially, the only feedback path from the plate to the grid of the tube must be through the crystal; therefore, the presence of the grid to plate capacitance which exists in a triode makes the triode unsuitable for this application. In addition, a tube having a very high g_m is desirable, and, in general, pentodes have higher g_m's than triodes.

The preceding analysis is concerned only with the pertinent a-c circuit elements; therefore, no attempt has been made to show the methods by which the d-c biases are applied to the vacuum tube. It is proposed to use the non-linear characteristics of the vacuum tube to limit the amplitude of oscillations. This requires that the tube be operated Class C with a relatively high control grid bias developed with a resistor and capacitor rectifying circuit. This method of obtaining bias is well known and the circuit is shown in Fig. 16. The

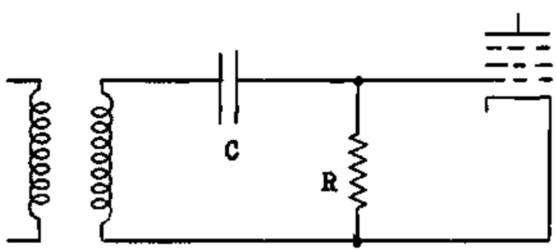

Fig. 16. Self Bias Rectifying Circuit.

value of the capacitor should be large enough that it introduces negligible reactance into the circuit. The value for R is most easily determined by experiment. However, it is noted that increasing R usually decreases the amplitude of oscillation, and that too high an RC product may produce an intermittent or self-blocking condition of oscillation. Obviously, too low an RC product will prevent oscillations completely. The plate-supply current is most easily applied by by-passing the low side of the plate circuit transformer to ground, and connecting the d-c source at this point. A practical oscillator is presented in Fig. 17.

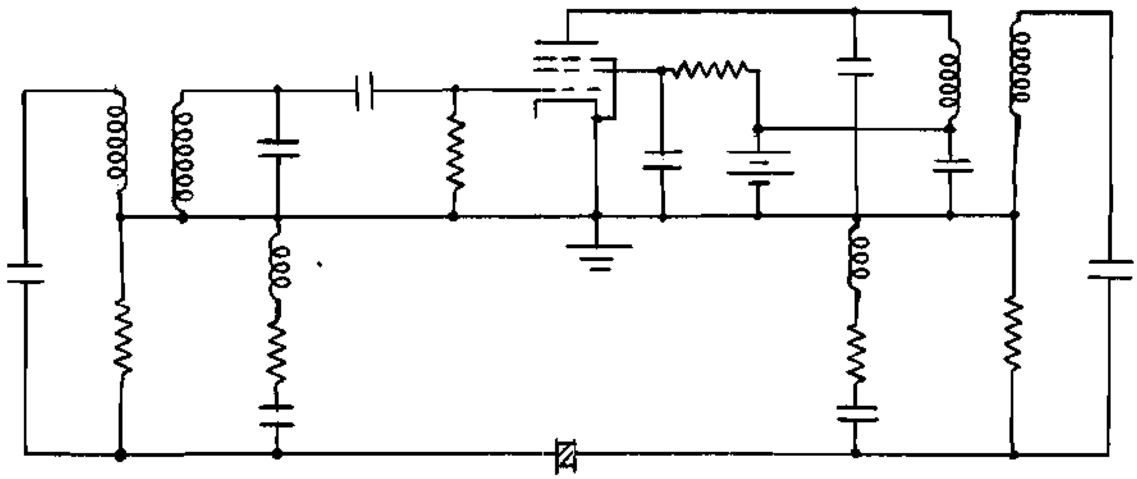

Fig. 17. Practical Oscillator Circuit.

The circuit was tested with hand-wound powdered-iron core, toroidal transformers. These transformers had a $k = 0.6$ which was apparently due to the relatively high transformation ratio required. Very stable continuous oscillations were observed at a frequency substantially equal to the series-resonant frequency of the crystal. Because of the relatively low coefficient of coupling, however, it was necessary to readjust the transformers and capacitors in order to operate with crystals having frequencies which differed by more than a few kilocycles from the original.

In concluding this discussion it is noted that the particular circuit which was developed is only one of many possible solutions to the original problem and that the limitations upon circuit performance are, at present, due to the transformers as constructed by present techniques.

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